AR 8/3 2010

ECON 4330 Spring 2008 Answers to home-work exercises

Exercise 1A

1) The simplest explanation is that the government budget is balanced every year, $T_t = G_t$. However, since the representative consumer has an infinite horizon, it is sufficient that the budget balances in present-value terms:

$$\sum_{s=t}^{\infty} T_s = \sum_{s=t}^{\infty} G_s$$

   This interpretation fits with that B_t is called “net foreign assets”, not “net assets of the private sector”, which indicates that Ricardian equivalence is assumed.

2) Saving one unit of goods from period s to period s+1 can increase consumption by (1+r) units in period s+1. Condition (3) states that along an optimal consumption plan the expected utility loss from a marginal increase in savings in one period should be equal to the expected utility gain that can be obtained next period. In other words, there should be no expected net gain in utility from saving more or less in any period.

3) Equation (4) says that the consumer is planning to have the same level of consumption in all periods. Actual consumption levels may of course change over time as new information arrives. However, no information that is available to the consumer at time *t* can be used to predict future consumption. The change in consumption from one period to the next is a random variable with mean zero.

   In order to get from (3) to (4) we need to assume that a) the utility function is quadratic and b) $\beta(1 + r) = 1$. The latter assumption means that the subjective discount rate implicit in the utility function is equal to the market interest rate *r*. If $\beta(1 + r)$ were different from 1, the consumer would plan for a trend growth in consumption. Without uncertainty $\beta(1 + r) = 1$ would be sufficent to get complete consumption smoothing. (In addition to the assumption explicitly mentioned in the question, I take for granted that the period utility function is concave, and that the interest rate is non-stochastic since it has no time index).

4) Total wealth is the sum of the value of claims on foreigners and the expected present value of the present and future income endowments after tax. (6) says that each year one should consume the annuity value of total wealth. The annuity value is the constant income stream that has the same present value as total wealth. In this case the annuity value happens to be equal to the interest rate times the value of total wealth at the end of last period. (6) is also identical to the consumption function we would have got in an identical model with full certainty, except that the actual values of future income are replaced by their mathematical expectations. The equation displays “certainty equivalence”.

Certainty equivalence is obtained by assuming quadratic utility. With quadratic utility marginal utility becomes negative for high consumption levels. This acts as a disincentive for saving. Also, marginal utility is finite when C=*0.* This makes negative consumption levels a possible outcome. Most actual consumer would probably show a more cautious behavior when facing income uncertainty, especially if the income path has low income levels to begin with. Still, some degree of consumption smoothing seems to be a characteristic of consumer behavior.

The assumption of no trend in consumption is difficult to uphold in a world where total output is growing.

Another assumption is that of a representative consumer who maximizes over an infinite horizon. This is based on that today's consumers incorporate the utility of their descendants when they maximize, and that transfers between the generations are possible in both directions. Other assumptions about intergenerational transfers may make consumption more dependant on current income.

5) In this case we can define the current account surplus as

$$CA_t = rB_t + Y_t - C_t - G_t$$

Since we are not asked about G, I am going to set it equal to zero for all t. After inserting from (6) we then get

$$CA_t = Y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \mathbf{E}_t(Y_s) = Y_t - \bar{Y}_t$$

where $\bar{Y}_t$, defined implicitly by the last equality, is the *permanent income*, the annuity value of all present and future expected incomes. A pure transitory shock would be a shock that increases Y_t but leaves all all future *Y's* unchanged. Since Y_t is a component of$\bar{Y}_t$, $\bar{Y}_t$ increases, but only marginally. Hence, CA_t increases (almost) one for one with Y_t as the consumer tries to spread out the extra income on an equal increase in consumption in all periods. A permanent shock would raise Y_t and $\bar{Y}_t$ by equal amounts. As we can see from the expression above, this has no effect on the current account.

Exercise 2

For questions 1-4 you can use that UIP in discrete time can be written:

$$1 + i_t = \frac{E_{t+1}}{E_t}\,(1 + i_{*,t})$$

You may think of the peso-country as the home country.

1. The exchange rate today is 10*(1.05/1.10) = 9.55 pesos/dollar
2. The expected exchange rate two years from now is $(1.04/1.05)^2$*10 = 9.81 pesos /dollar
3. See figure 1. Before the news arrives at time 0 the exchange rate is 10. In year 3 we suppose that eveything is back to normal, including the exchange rate. In years 1 and 2 the peso interest rate is higher, and hence a gradual depreciation of the peso is expceted. In year 0 both interest are equal and no depreciation is expected. As long as interest rates are finite (and UIP holds), there can never be expected jumps in the exchange rate. The only jump is at time zero when the news arrive and the exchange rate immediately appreciates.
4. See figure 2. In year 4 everything is back to normal. During year 3 the peso interest rate is lower and the peso is expected to appreciate. In year 2 both countries are in recession. We assume they then have the same interest rates, and expected depreciation is zero. During year 1 the dollar interest rate is lower and the peso is ecpected to depreciate. If the amplitudes of the interest rate movements in the two countries are the same, there will be no initial jump in the exchange rate. Remark: It is not obvious that it is rational to expect that the exchange rate returns to its previous level after a recession like this.
5. Sorry, there is a misprint in the question. It was the dollar interest rate that was meant to be 5 per cent per year. If that is assumed the peso interest is [(10/9)*1.05 – 1] = 17 %

In question 5 you use

$$1 + i_t = \frac{F_{t+1}}{E_t}\,(1 + i_{*,t})$$

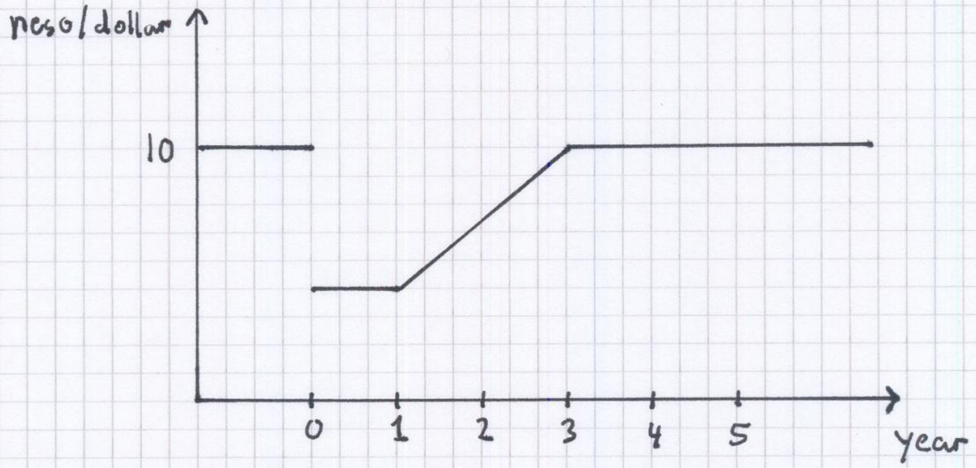

Figure 1

Figure 2